Ongoing efforts to promote students' engagement in defining and conjecturing

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Welcome! **Two requests!**:

1. Sit on the side of the room depending on which topic you'll likely want to discuss
   ← Promoting equitable participation — Integrating science into math tasks →

2. It would be awesome if at least one person at your table would be willing to use a QR code throughout the talk to add your group's ideas our Padlet

Motivation

"You're not being handed a finished package and being told to just take it as is. You're building it. It's the difference between being given an assembled Lego set and building it yourself..."

–ASPIRE student
White woman

This quote is from a student who was commenting about their experience in a course that was specifically designed to engage students in defining, conjecturing, and proving.

What do you notice about her quote?
What do you wonder about her experiences in math classes?

Turn and talk to your neighbor! Use the QR code to add your group's noticings/wonderings to our Padlet.

Past(ish)

ASPIRE in Math

Not our Padlet! But where you can go to get more information about our project
(NSF DUE 1916490)

We created modular inquiry-oriented introduction-to-proofs curriculum & instructor support materials

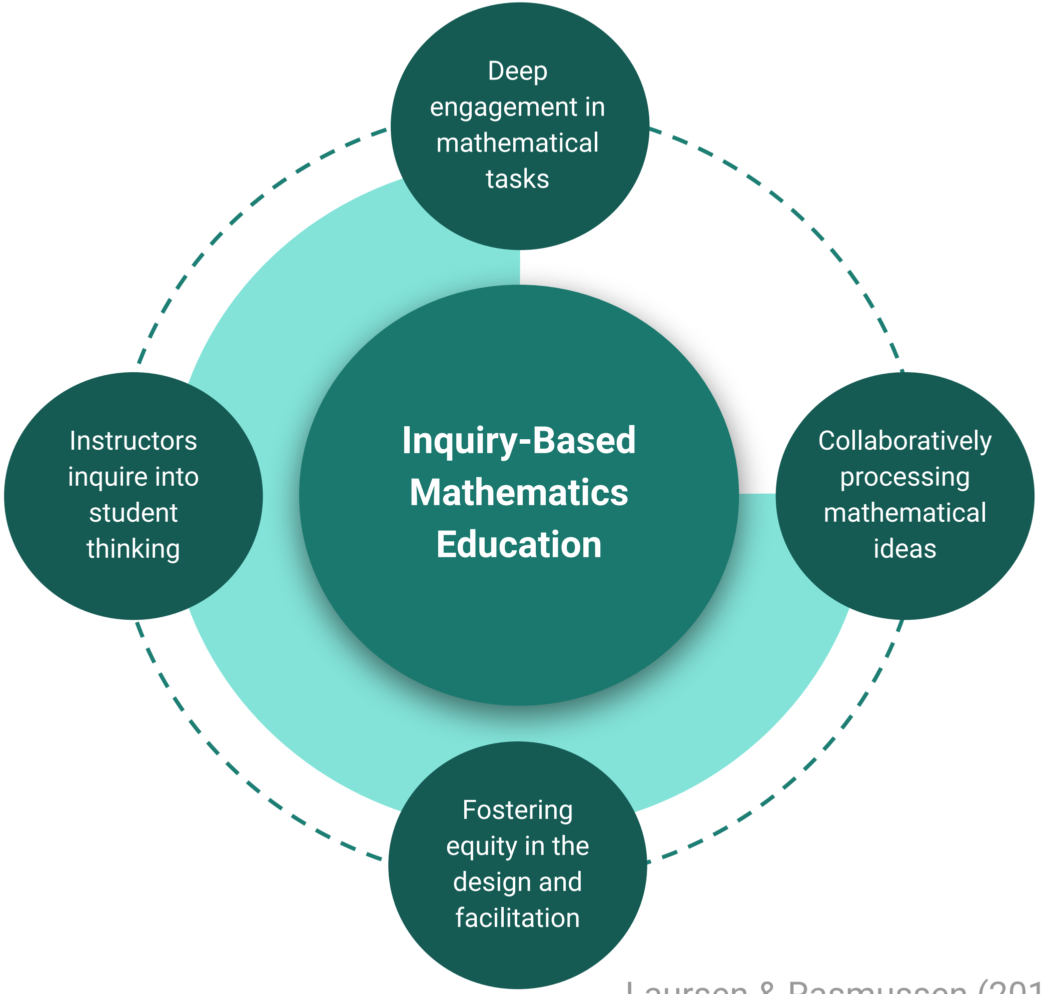
A big goal is to support students with the transition to advanced mathematics

We have been working with university and community college students

Inquiry-Oriented Instruction (IOI)

Research-based curriculum created using design-based research methods

Guided by the instructional design theory of Realistic Mathematics Education



Laursen & Rasmussen (2019)

Realistic Mathematics Education (RME)

Mathematics is a human activity

Guided Reinvention: students are guided (by the instructor) to create mathematics from their informal ideas

- Supports progressive mathematizing

Students work within a context that is *experientially real* to them

- The problem is accessible and meaningful to them
- Can be created through historical examinations or examinations of students' informal solution strategies

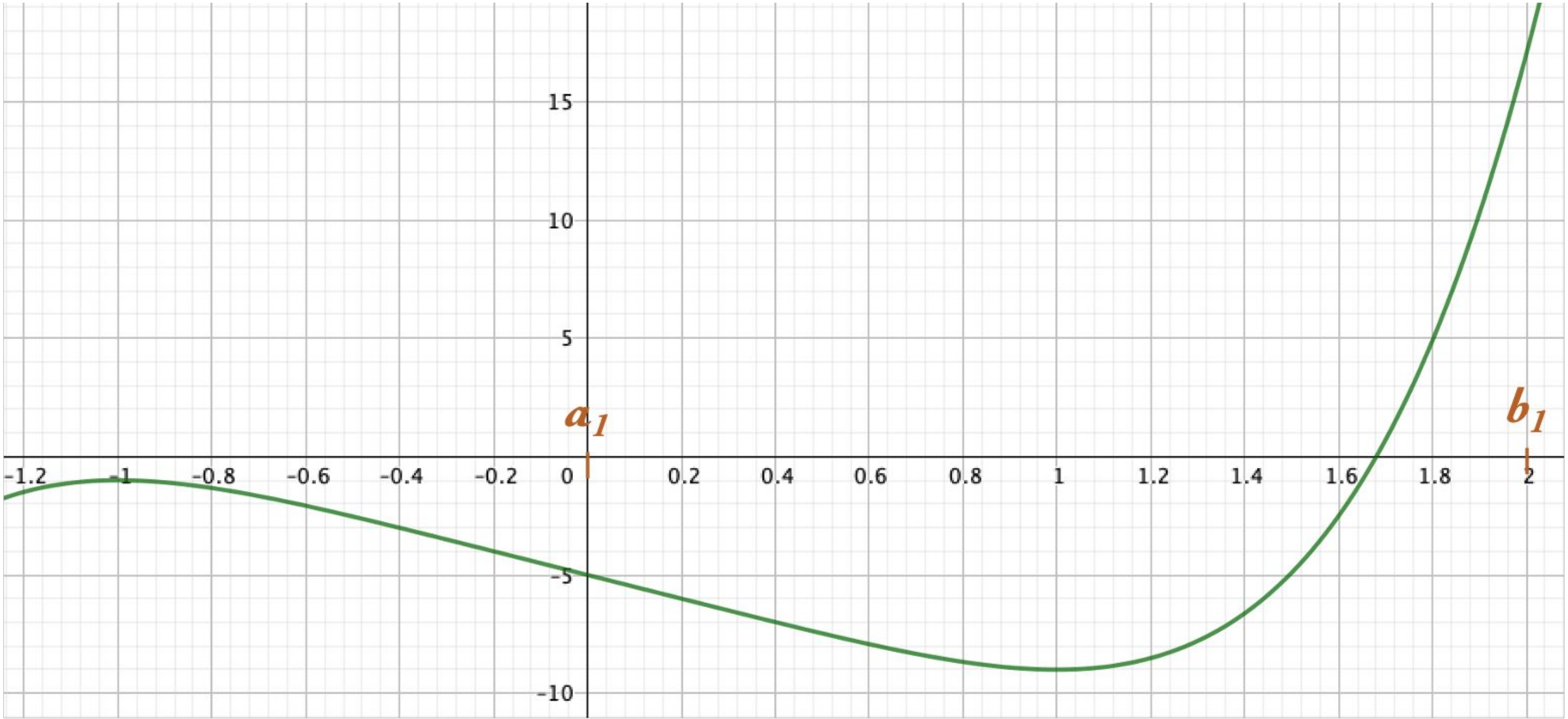
(Gravemeijer, 1999; Larsen, 2018; Rasmussen & King, 2000)

A motivational task from the Real Analysis module

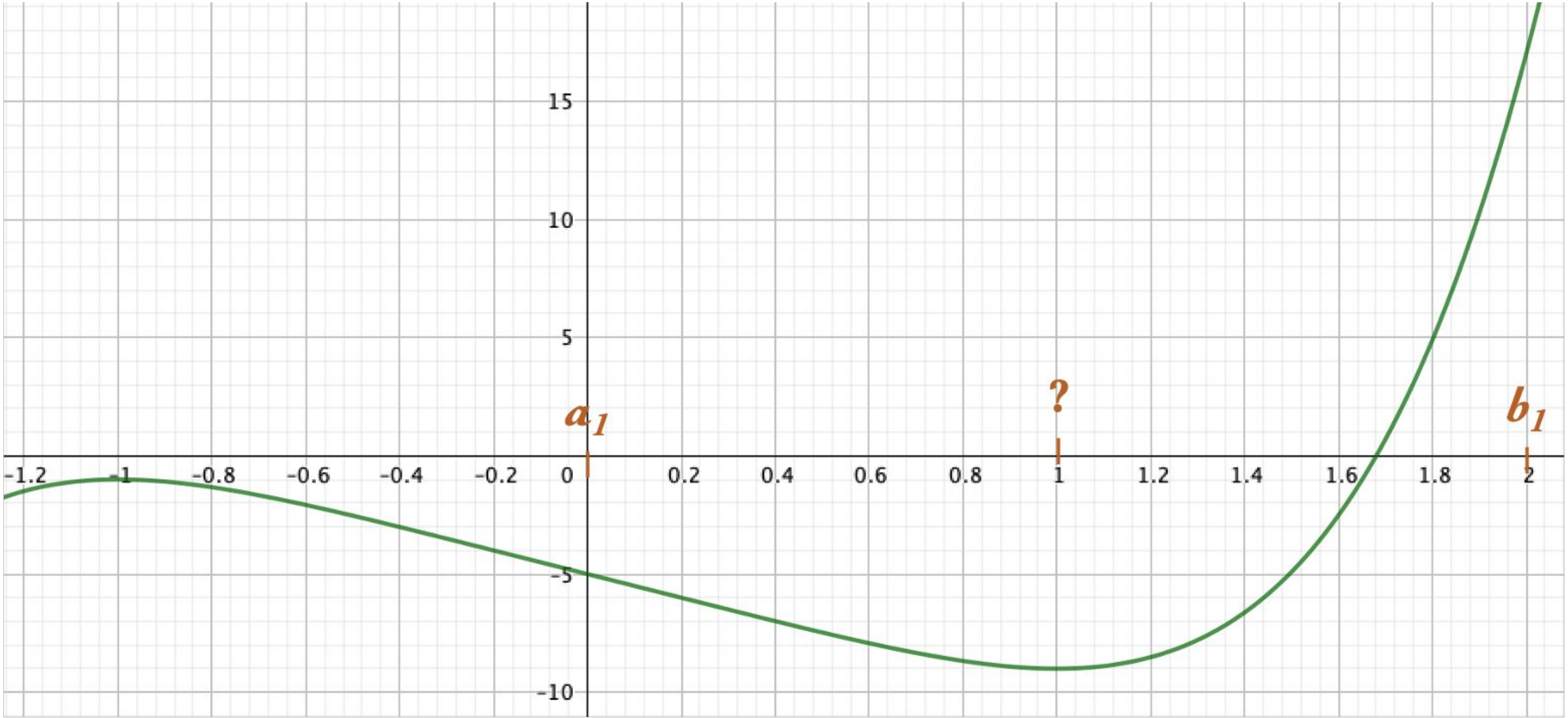
1. Students are supported to conjecture that every odd degree polynomial will have at least one (real) root
2. Students reinvent a method for approximating a root
3. Students notice sequences and some sequence properties from this approximation method

One approximation method (the "bisection method") goes like this...

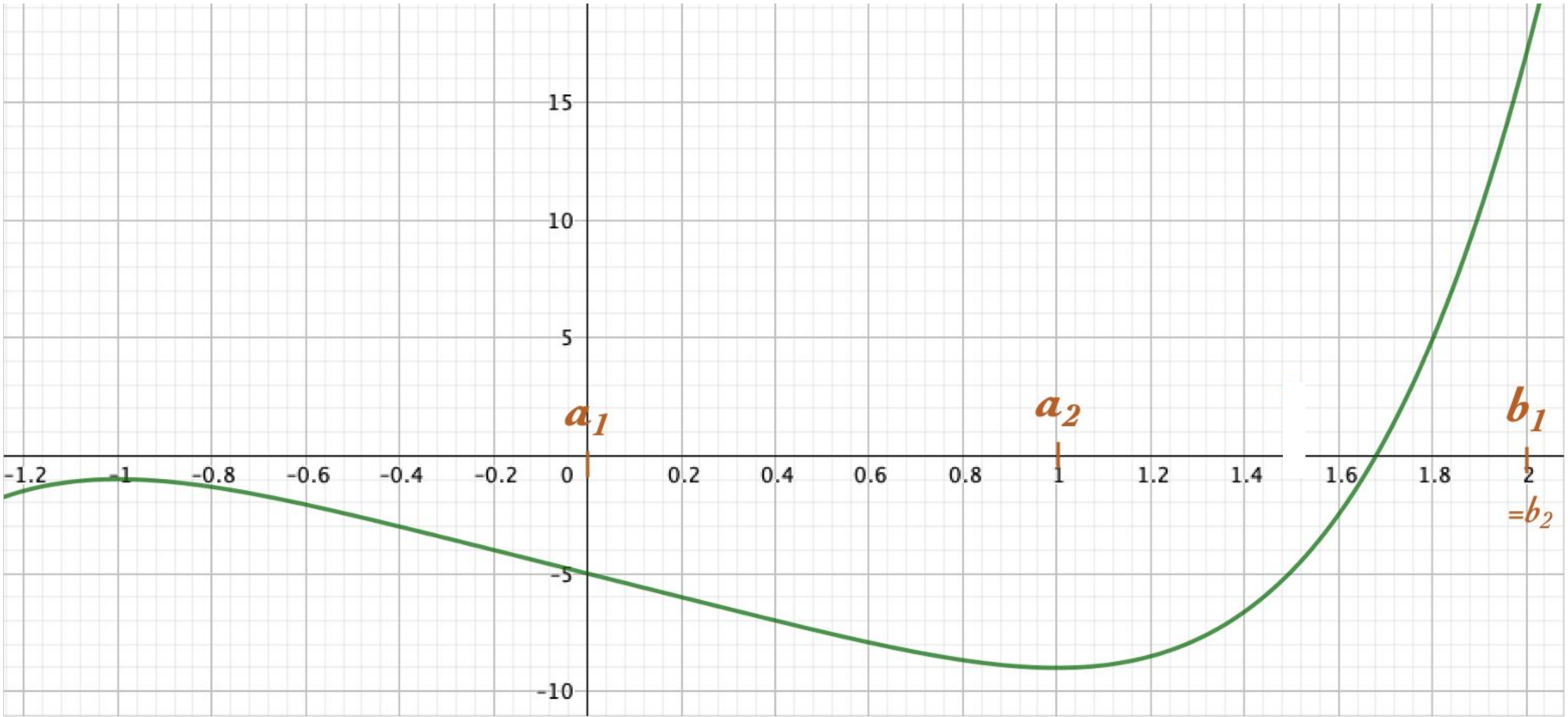
I take two inputs, a_1 and b_1, that map to opposite signs



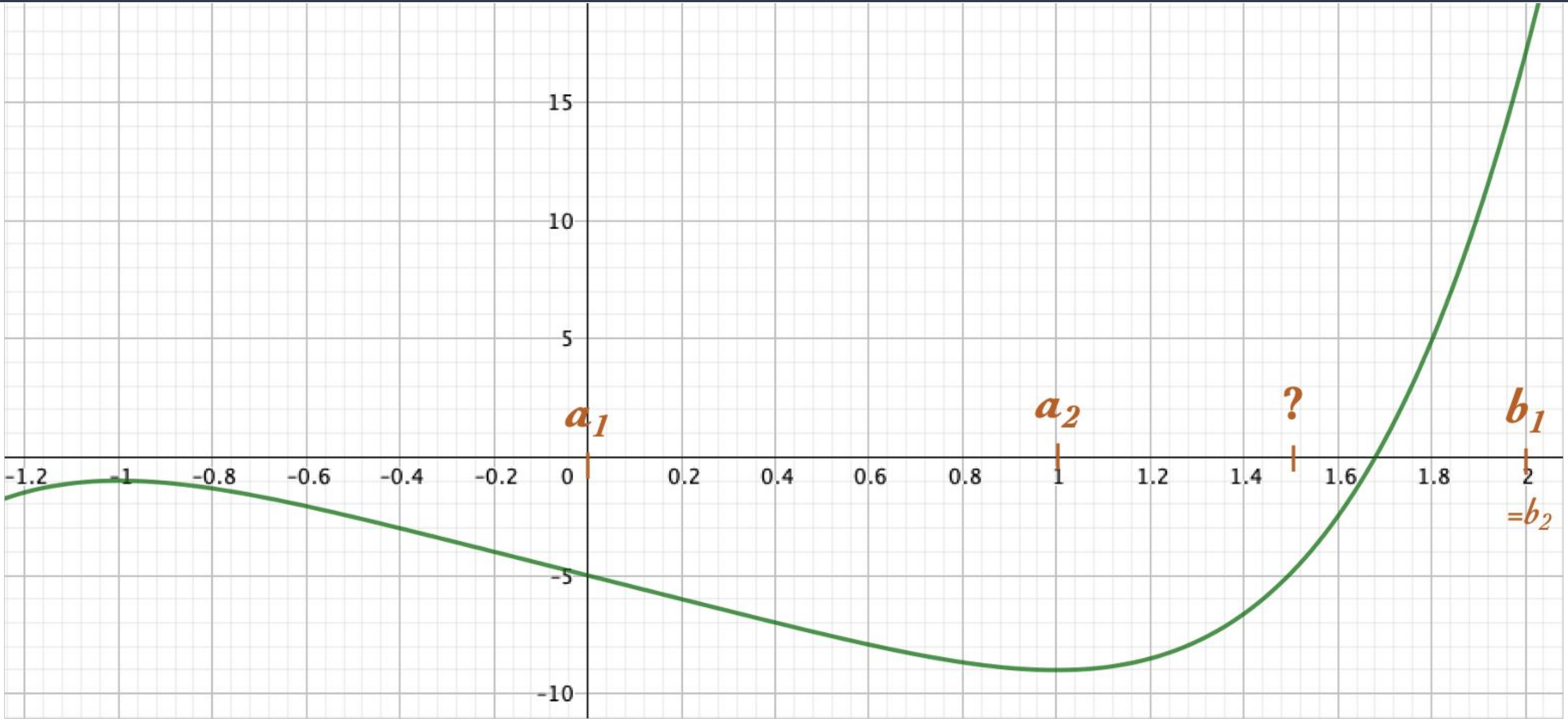
Then I find the midpoint – and ask myself:
Does this input map to a positive or negative value?



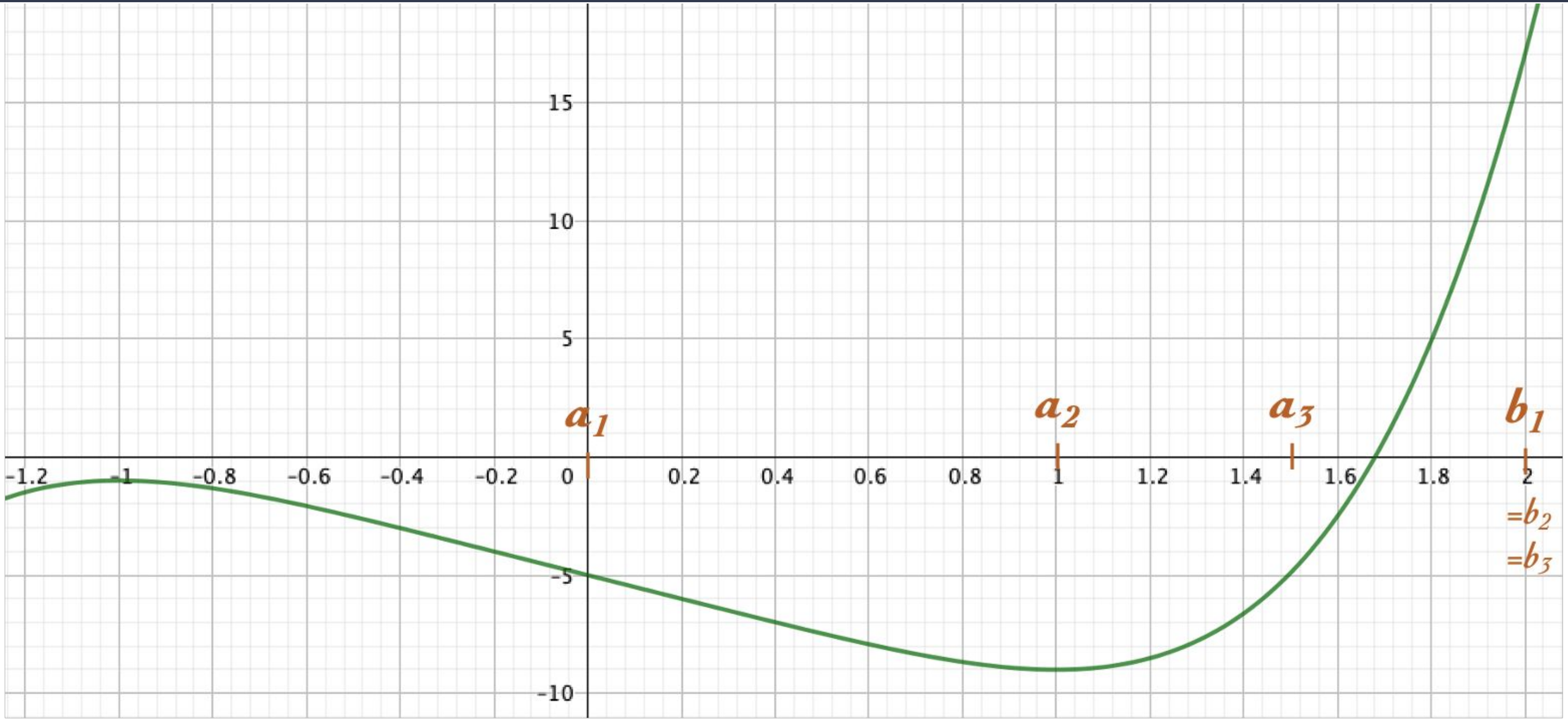
Since this midpoint maps to a negative value, I call it a_2 and b_2=b_1. Now the root-candidate is between a_2 and b_2



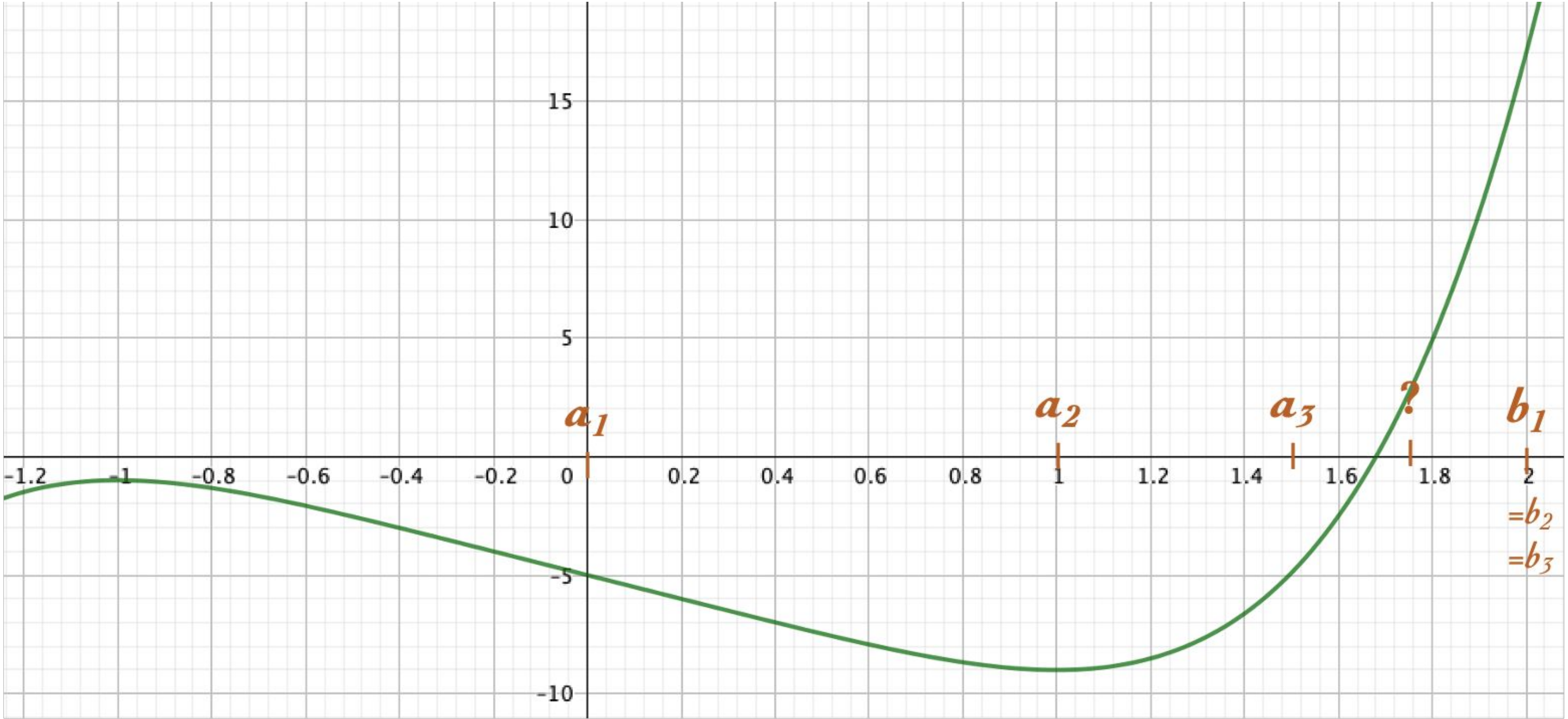
I find the new midpoint and again ask myself: Does this input map to a positive or negative value?



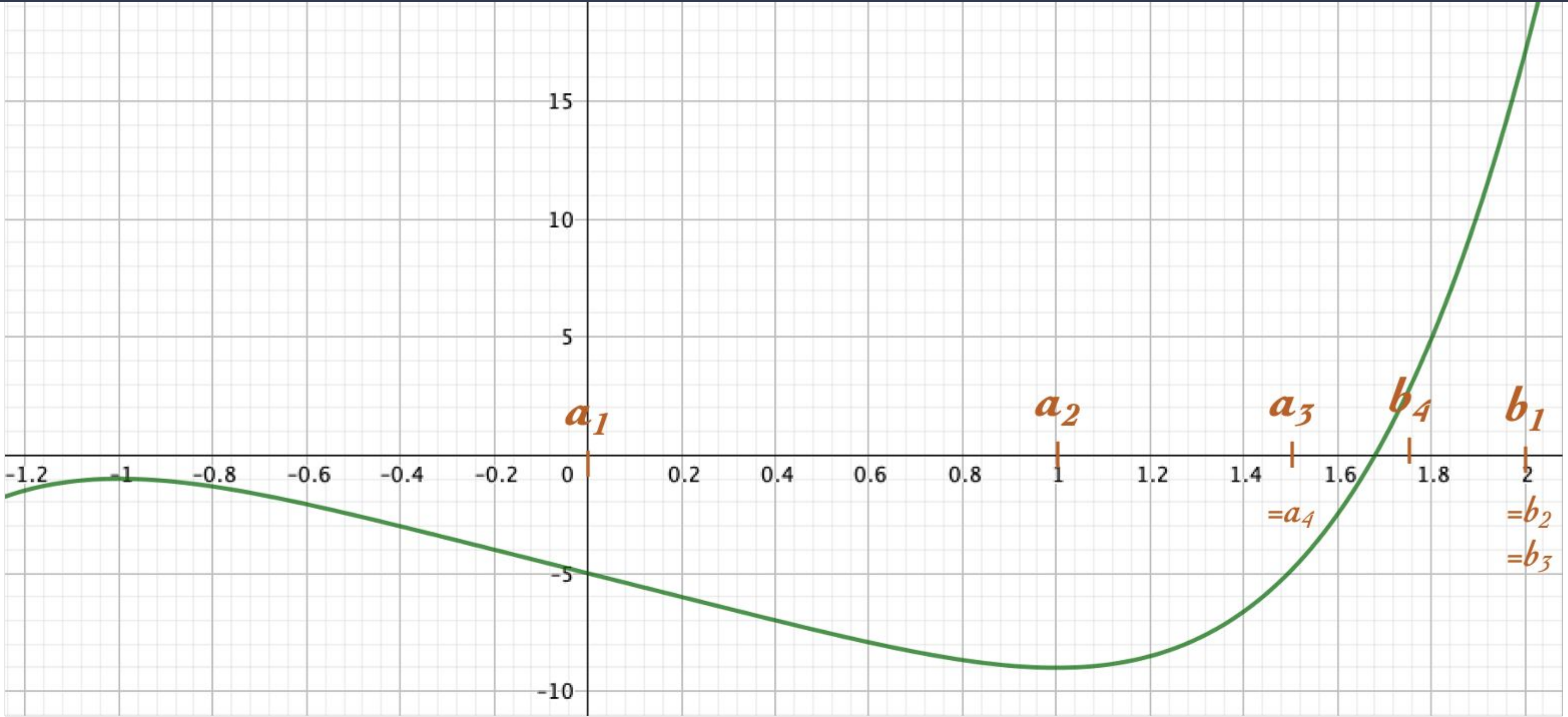
This midpoint also maps to a negative value. So I label it a_3 and b_3=b_2



I find the new midpoint and ask myself:
Does this input map to a positive or negative value?

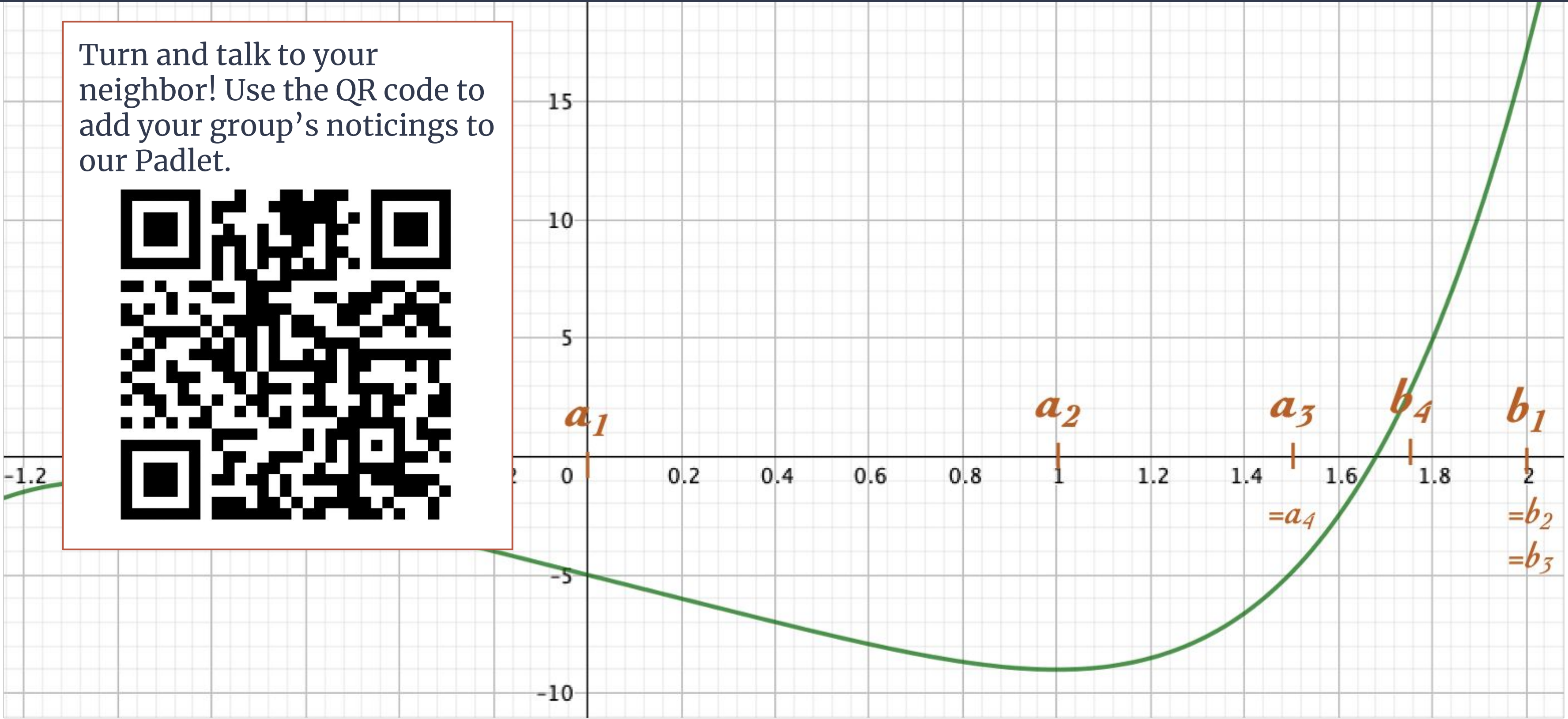


This midpoint maps to a positive value! So I label it b_4 and a_4=a_3



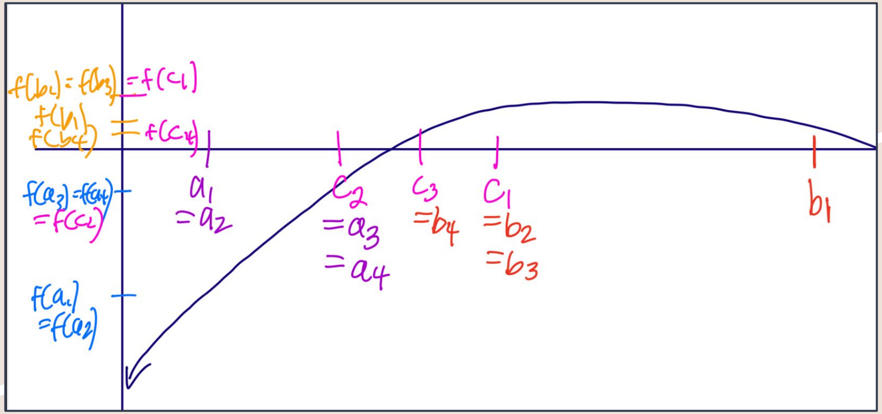
Imagine continuing the bisection method forever! What do you notice about the a_ns and the b_ns ???

Turn and talk to your neighbor! Use the QR code to add your group's noticings to our Padlet.



The bisection method…

1. Is experientially real for undergrad students because it's accessible: these students have prior experience with polynomials and finding their roots
2. Is rooted in a historical context: motivated by Cauchy's strategy for proving the IVT



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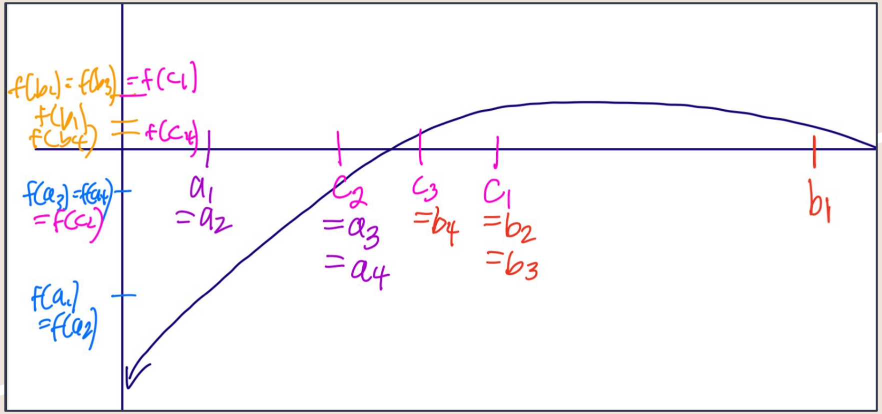


Among other things, it evokes informal ideas about…

- Sequences (as functions mapping the natural numbers to the reals)
- Increasing/decreasing sequences
- Bounded above/below sequences
- Converging sequences

(Larsen et al., 2022)

Wondering:

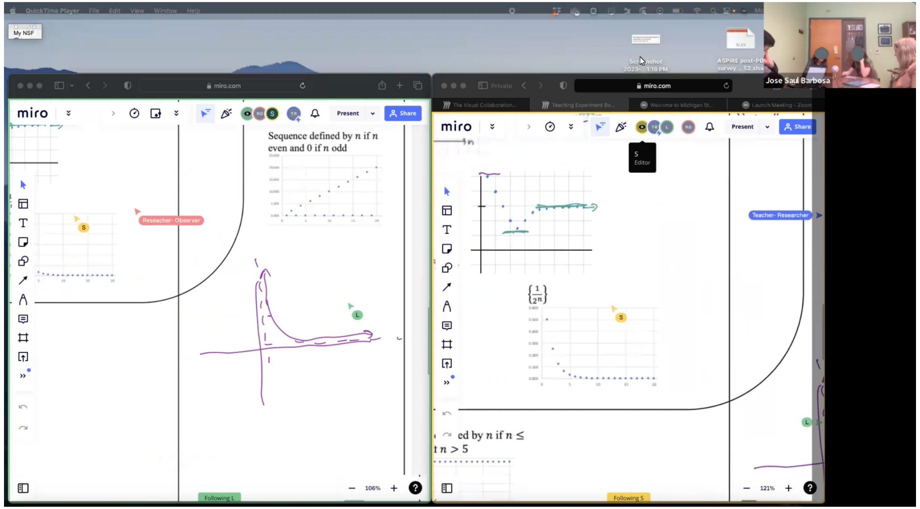
How can we support students to engage in this creative activity earlier in their college experience?

Present

Currently conducting a design experiment with two students, Lara and Stella

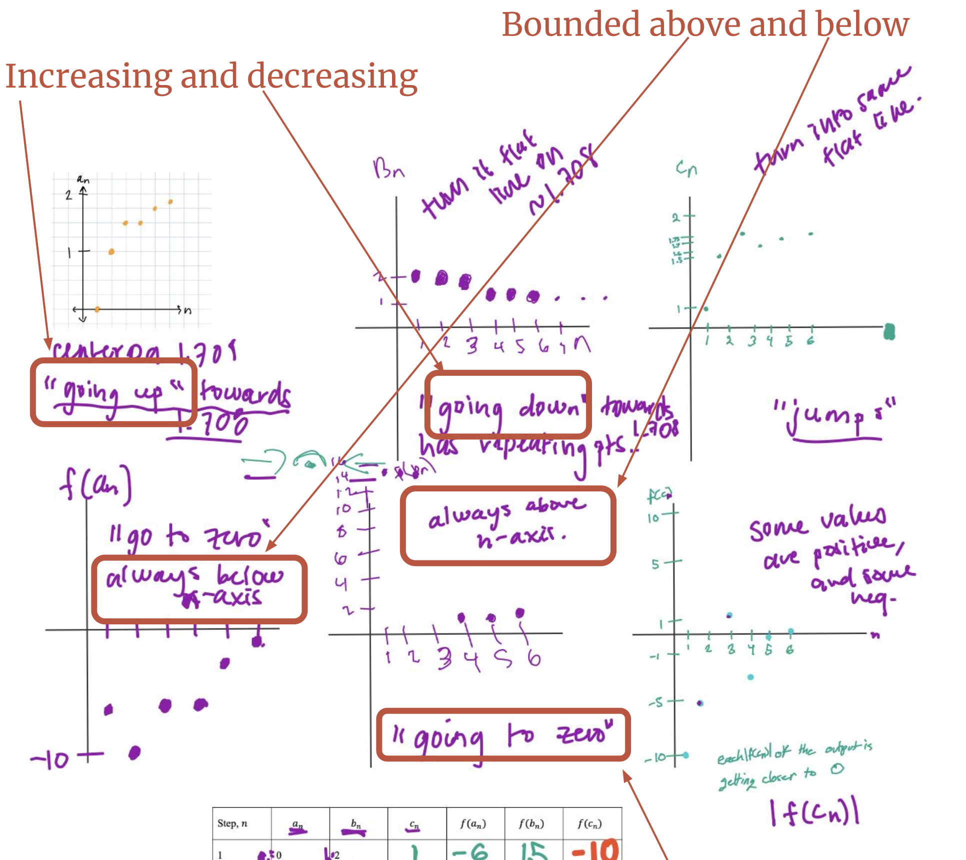
Goal: adapt and extend ASPIRE materials for Calc 2 course

Data is screen recording of us working on a collaborative white board (Miro)



Stella and Lara made sense of the bisection method and used it to approximate a root of a 5th degree polynomial by going through 6 steps of the method. Then, they considered continuing the method forever.

We identified 6 objects (which we termed “sequences”) and Stella and Lara discussed some informal ideas about some different properties

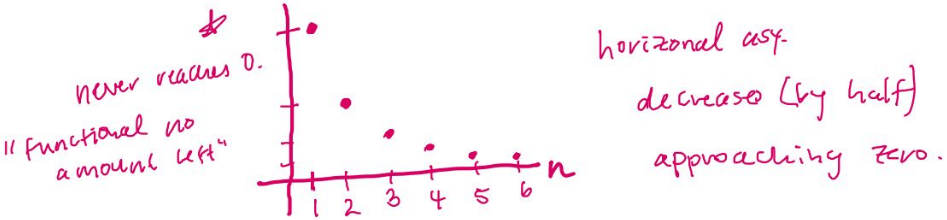


Step, n	a_n	b_n	c_n	$f(a_n)$	$f(b_n)$	$f(c_n)$
1	a_1: 0	b_1: 2	1	-6	15	-10
2	a_2: 1	b_2: 2	1.5	-10	15	-5.9
3	1.5	2	1.75	-5.9	15	1.7
4	1.5	1.75	1.625	-5.9	1.7	-2.8
5	1.625	1.75	1.6875	-2.3	1.7	-0.3
6	1.6875	1.75	1.71625	-.3	1.7	0.3

Convergence

We then asked Stella and Lara to generate more examples of sequences that were in some way different from their previous ones.

Lara provided an example of a “half life” sequence



Then they suggested *sin(x)* and *cos(x)* (with the domain of the real numbers) as more examples. We requested that they “list” the terms, which revealed that this was not possible and thus these functions were non-examples.

Writing and Refining a Definition for “sequence”

keep "stepping"

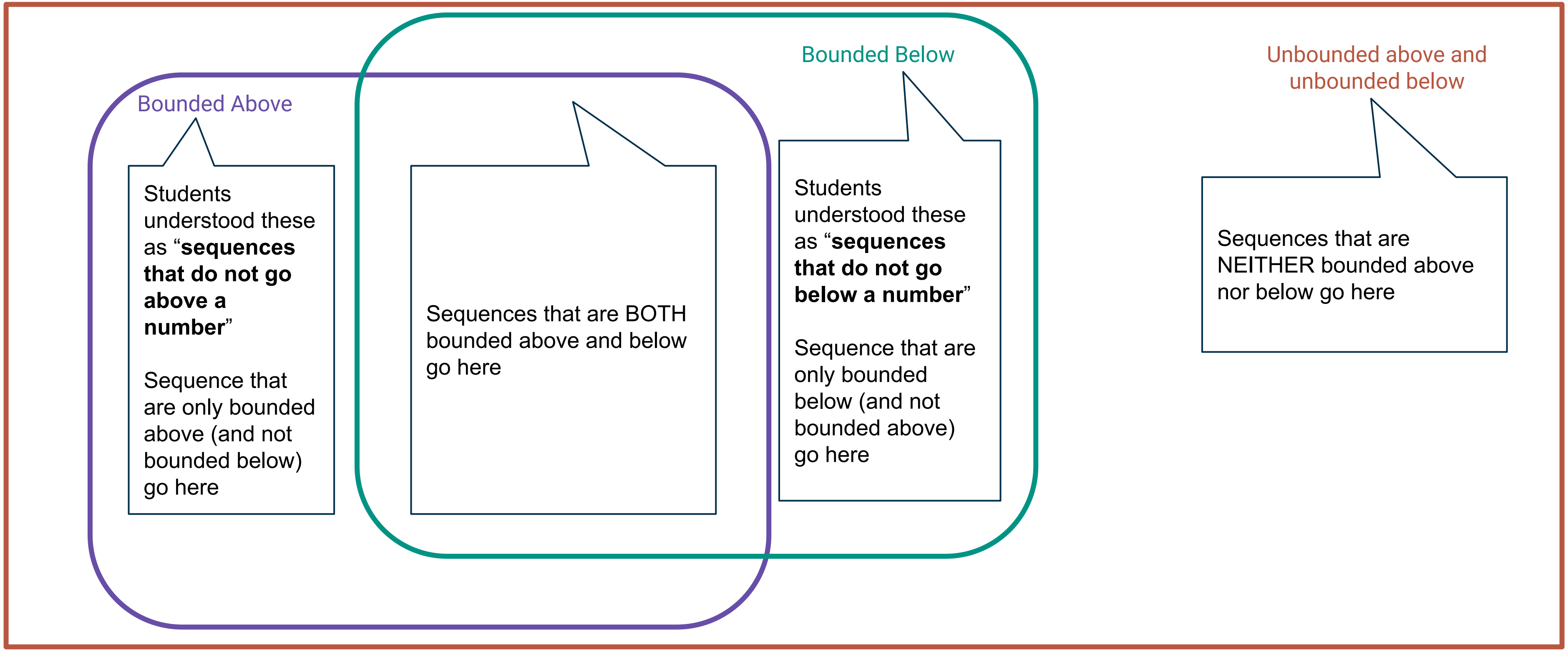
"rule"

A sequence $\{a_n\}$ is…	A sequence $\{a_n\}$ is…
a sequence is a continuous list that follows a certain mathematical relationship through all values of x.	an infinite list of numbers that follows some trend that can be modeled by a function

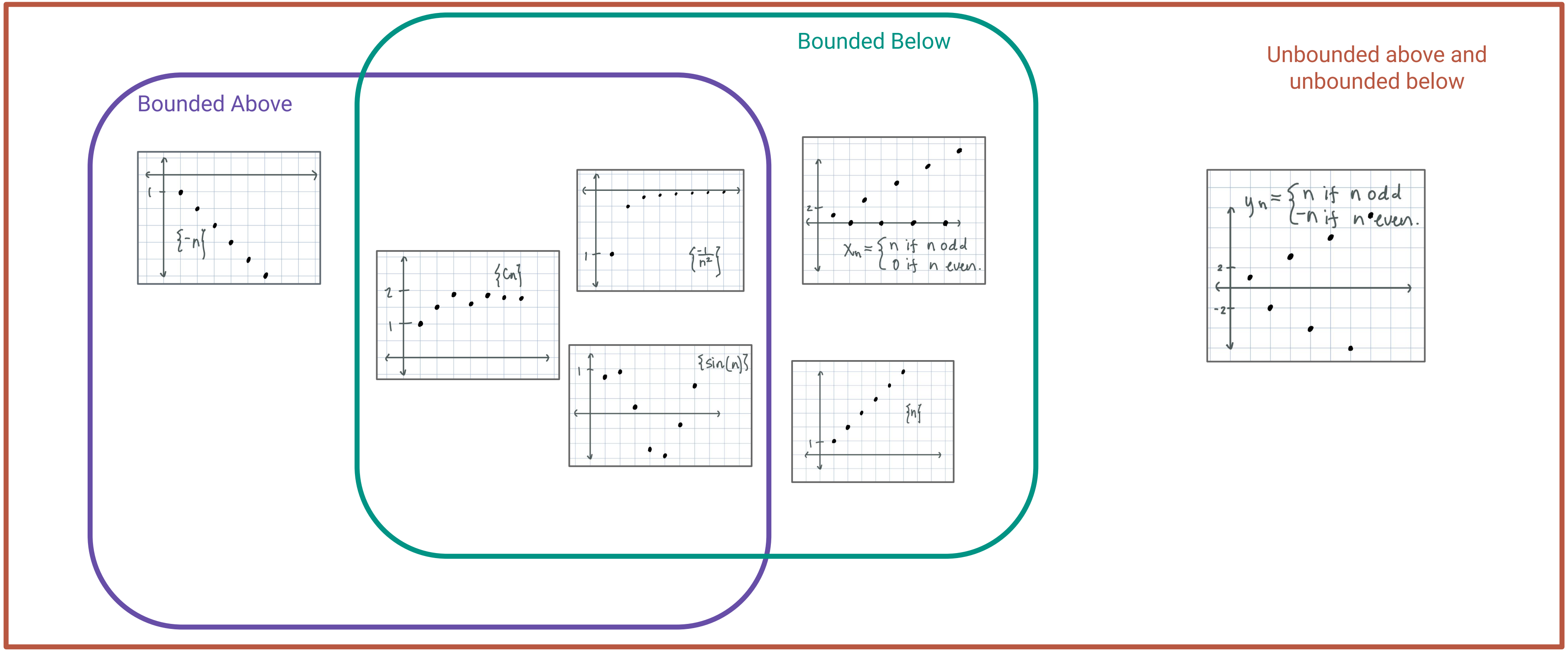
A sequence $\{a_n\}$ is … a list of only natural numbers as inputs, where there is always an output

and the relationship can be defined as some mathematical function

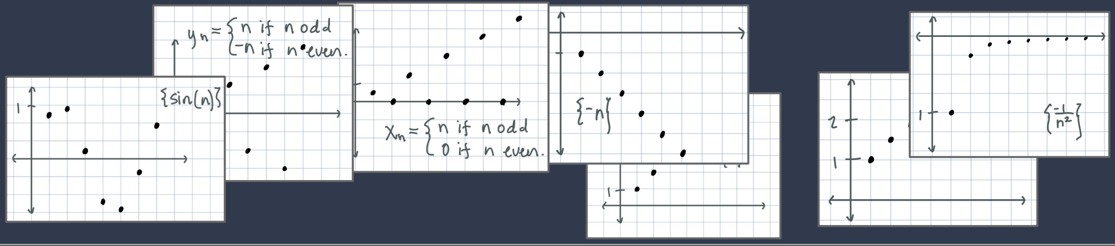
You have some sequences on your table! Sort them on your "map" based on descriptions of students' informal ideas:

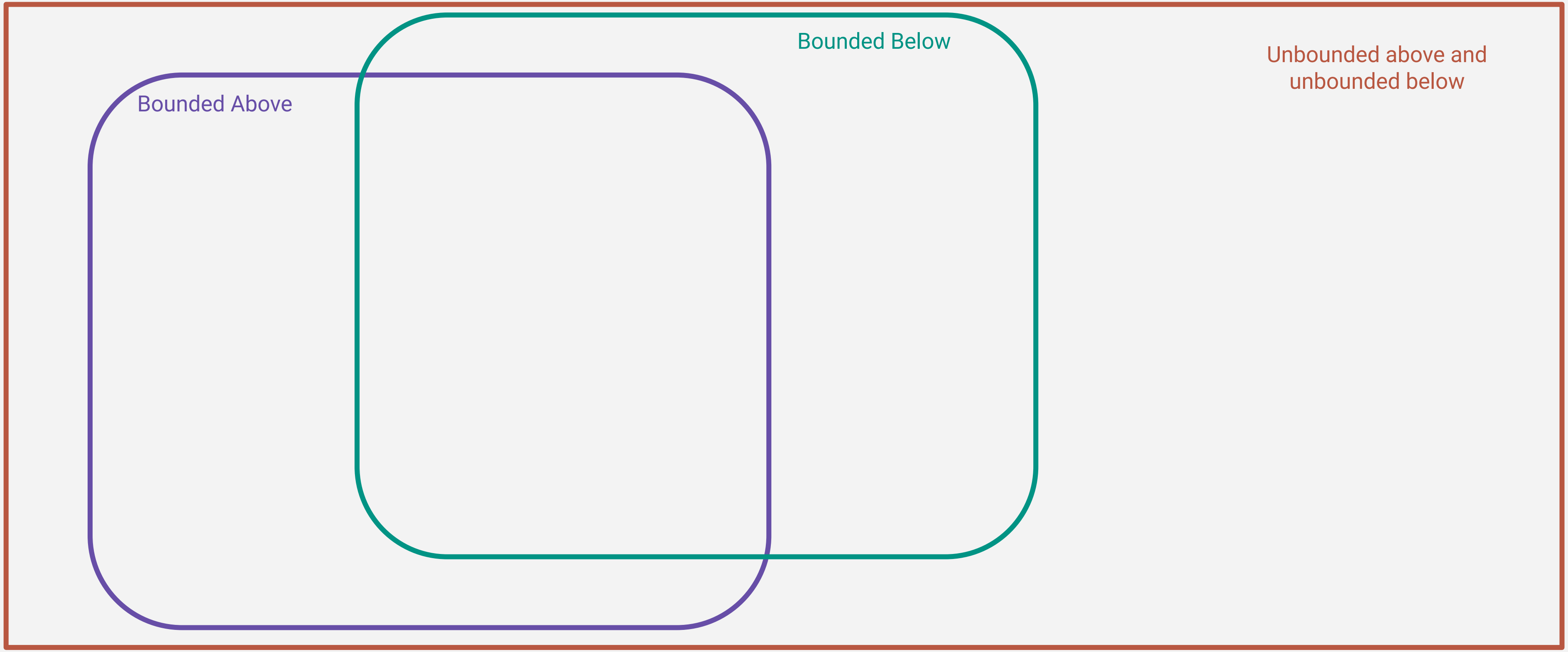


Lara and Stella did a similar thing (with more sequences):



Zoom attendees' map!:





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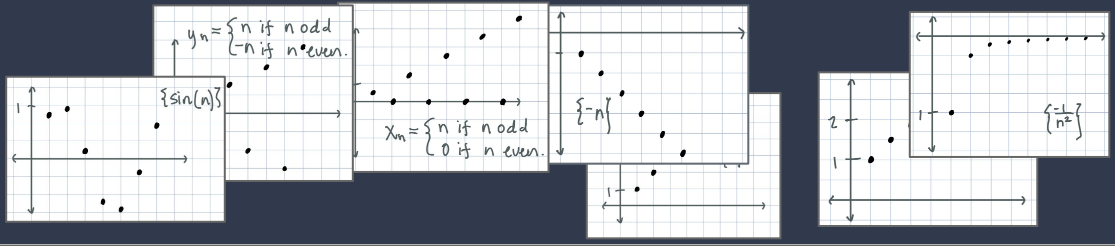
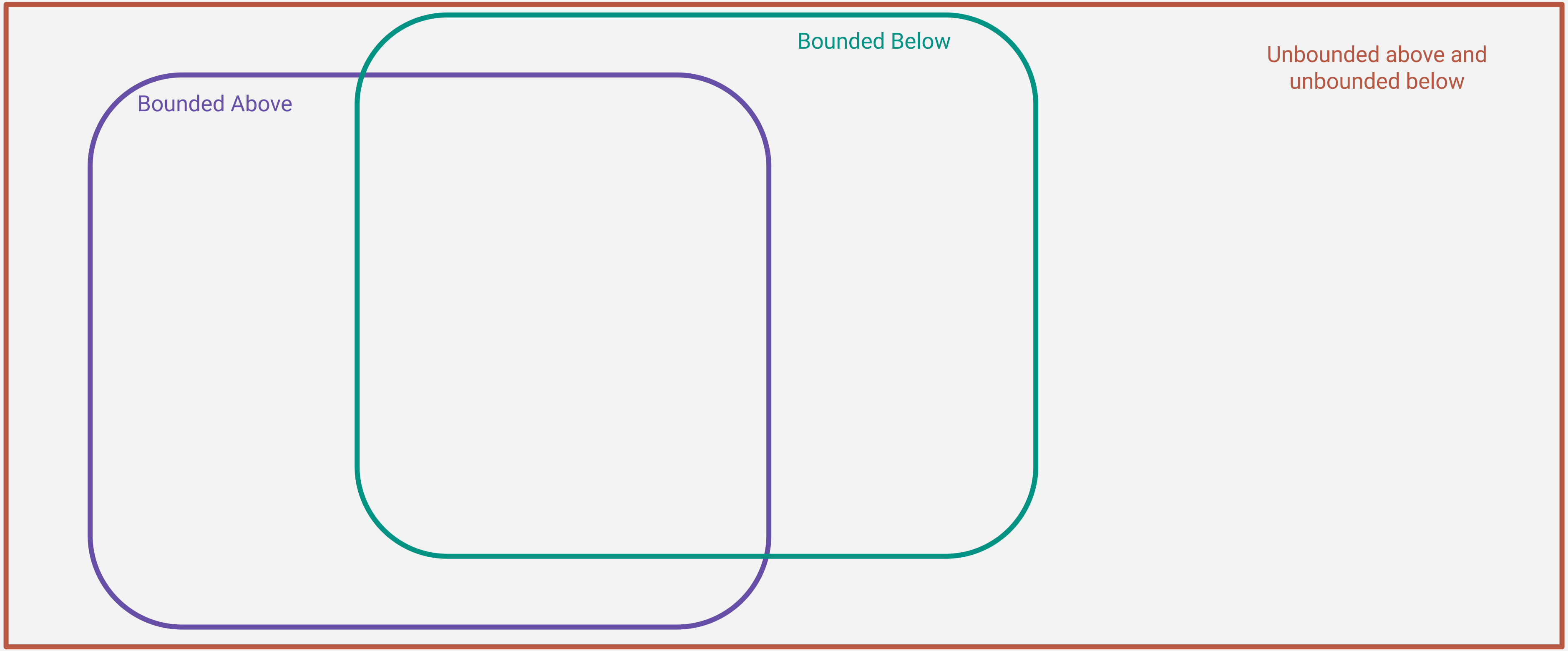




Where does “increasing” belong?

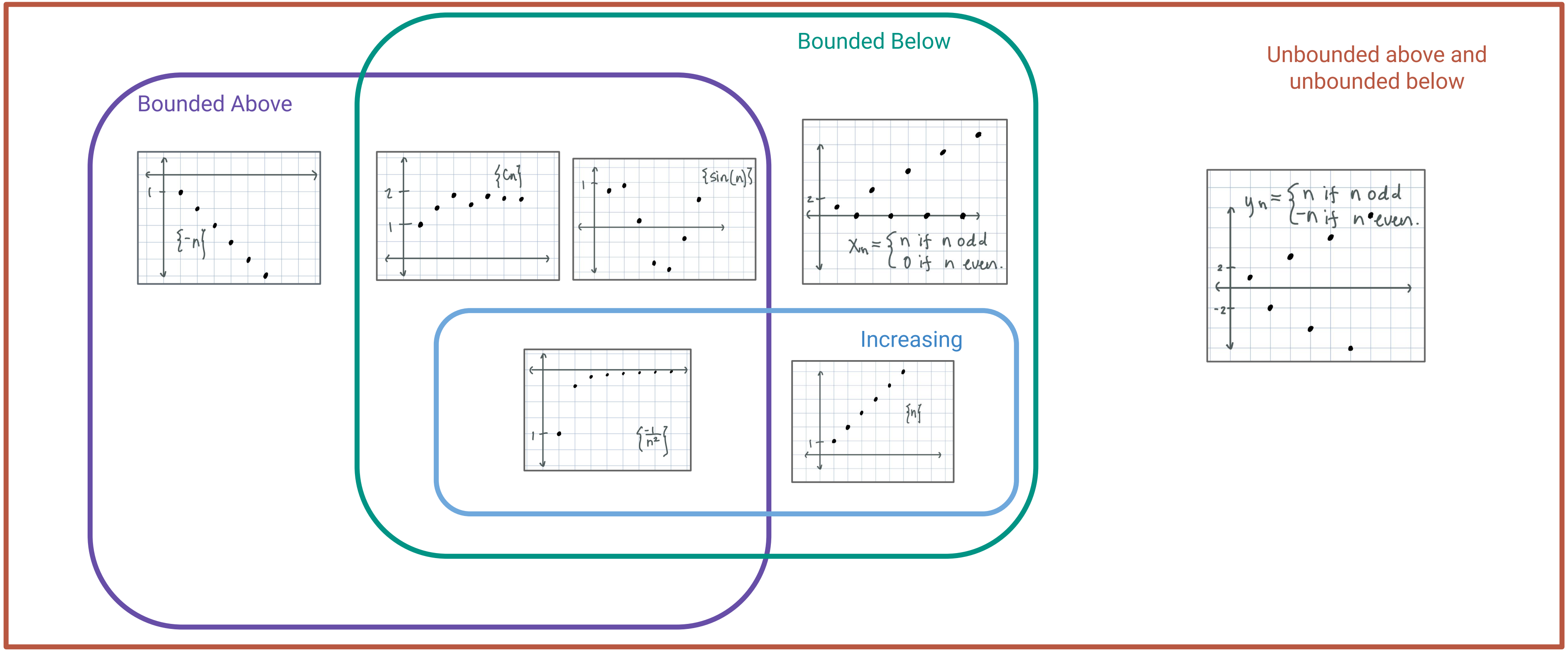
While they were sorting, Stella conjectured: “*increasing means unbounded above.*” By increasing she meant sequences that fit their following definition (defined previously in the experiment), which seems to be conceptually equivalent to the definition in our Stewart calculus textbook (some might call this “strictly increasing”)

> A sequence is increasing if...
> all of the points must have a higher value than the previous input value.

And so we explored Stella’s conjecture by asking them to add an increasing-bubble to their map.

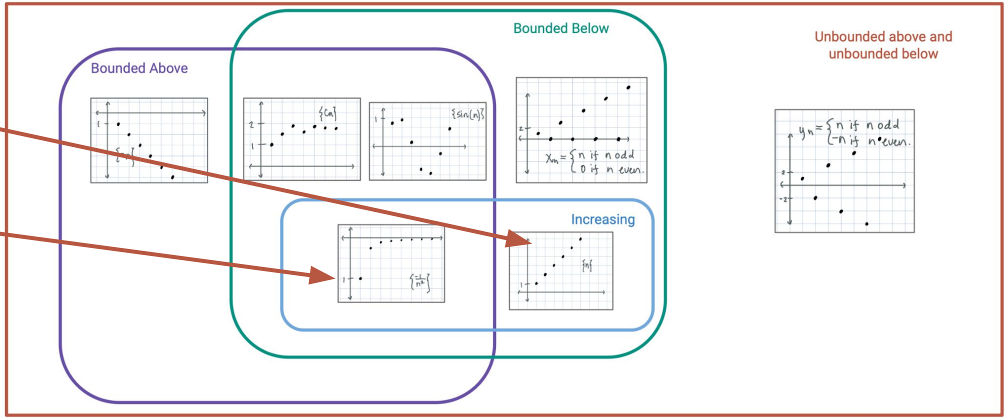
Let’s do the same!: **Draw a bubble for increasing sequences on your map and re-sort the sequences!**

Lara and Stella put it here!



Notice that with this the students have enough evidence (a counter example) to show that the following are *false*:

- All increasing sequences are unbounded above
- All unbounded above sequences are increasing



This map suggests that the students also conjecture that the following is *true*:

- All increasing sequences are bounded below

We are continuing to support Lara and Stella to engage in defining, conjecturing, and justifying.

Currently we are doing this with sequences and eventually we will with series.

Anticipated student sorting:

Series Convergence	Series Divergence	
$\sum \frac{-1}{n^2}$	$\sum n$	$\sum x_n$
	$\sum -n$	$\sum c_n$
	$\sum \sin(n)$	$\sum y_n$

Future

Looking ahead:

1. In thinking about scaling up our task to the whole class setting, we would like to redesign tasks to optimize for equitable participation

2. In thinking about designing more tasks, we would like to find motivational science contexts that elicit informal mathematical ideas that are typically covered in Lyman Briggs math classes

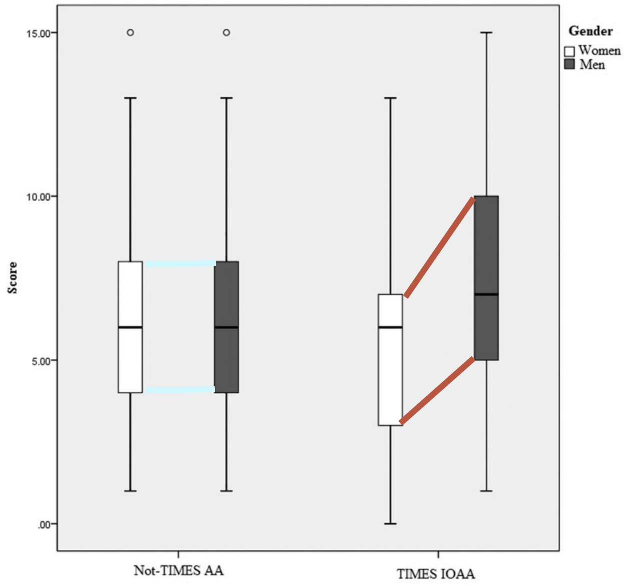
More active ≠ More equitable

Recent literature has documented in inquiry-based courses, students with marginalized identities in mathematics may:

- Score lower on achievement tests than their counterparts
- Experience exclusion
- Experience explicit microaggressions
- Have fewer opportunities to develop positive mathematical identities

1. Task redesign to optimize for equitable participation

Gender Comparisons of Student GTCA Performance



Brown, 2018; Ellis & Alzaga Elizondo, 2023; Ernest et al., 2019; Johnson et al., 2020, p. 511

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Lesson learned:
Inequities can be amplified if there are no intentional ways to mitigate them

1. Task redesign to optimize for equitable participation

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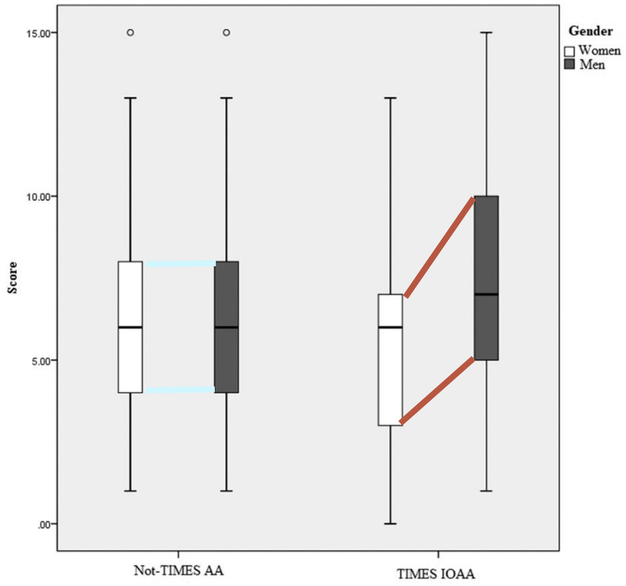

Brown, 2018; Ellis & Alzaga Elizondo, 2023; Ernest et al., 2019; Johnson et al., 2020, p. 511

We plan to adapt the task themselves so that we optimize for students' equitable participation*. So far, we plan to:

- Incorporate ideas from complex instruction (e.g., building in meaningful group roles into the task design)
- Incorporate revision structures into the tasks to support a culture of rough draft thinking (e.g., building in think-pair-share structures into the tasks)

1. Task redesign to optimize for equitable participation

*Students with marginalized identities in mathematics receive at least the proportional share of opportunities to contribute to the task at hand

Cohen et al., 1999; Featherstone et al., 2011; Jansen, 2020; Reinholz et al., 2022

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1. ## Task redesign to optimize for equitable participation

*Students with marginalized identities in mathematics receive at least the proportional share of opportunities to contribute to the task at hand

Caveat: We don't think this is going to solve the problem! The instructors' role is HUGE. But we want to set instructors up for success as much as we can from a task design perspective.

Cohen et al., 1999; Featherstone et al., 2011; Jansen, 2020; Reinholz et al., 2022

In thinking about Lyman Briggs students' interests, we want to design initial tasks (like the Bisection Method task) situated in science phenomena.

2. Motivational science contexts

Warren Christensen (from North Dakota State University) gave a plenary at 2023 RUME:

"Designing genuine interdisciplinary research projects and tasks is not trivial."

In thinking about Lyman Briggs students' interests, we want to design initial tasks (like the Bisection Method task) situated in science phenomena.

Looking for collaborators!

The motivational science context would be…

- "Experientially real" for students
- Elicit *informal* ideas about mathematical ideas that we cover (in Functions & Trig, Calc 1, Calc 2, or Calc 3)

2. Motivational science contexts

Warren Christensen (from North Dakota State University) gave a plenary at 2023 RUME:

"Designing genuine interdisciplinary research projects and tasks is not trivial."

"Mathematicians are bad a writing Physics problems!"

Group Discussion

Turn and talk to your neighbor! Use the QR code to add your group's ideas to the Padlet. You can also post questions you have for us on the Padlet!

Task design to optimize for equitable participation:

- What experiences (teaching, research) do you have in promoting equitable participation in the way you write/implement tasks? What have you learned from these experiences?
- Do you have suggestions for theory that could further guide our *task design* to promote equitable participation?

Motivational science contexts:

- What experiences (teaching, research) do you have in interdisciplinary task design? What have you learned from these experiences?
- What are some science phenomena that elicit informal ideas about mathematics concepts that we cover in Lyman Briggs (this could be sequences/sequence properties, series, real-valued functions and their properties, derivatives, etc.)

Thank you!

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